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On Equivalence of Spin and Field Pictures of Lattice Systems

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We investigate the spin and field systems on a lattice connected by the Kac–Siegert transform. It is shown that the structures of corresponding theories are equivalent (in the sense of isomorphy of space of Gibbs states and order parameters). Using the idea of equivalence of spin and field pictures, we exhibit a class of lattice systems possessing infinitely uncountably many ground states. The systems of this type with infinite-range, slow-decaying interactions are expected to have a spin-glass phase transition.

KEY WORDS: Lattice spin systems; random site long-range interactions; Kac–Siegert transform; Gibbs states; order parameters; ground states.

INTRODUCTION

A complete description of spin systems with two point interactions at high temperatures has been given in ref. 13. The main idea of this work was as follows: First we use the Kac–Siegert transform to pass from a spin system (with discrete variables) to a lattice field system (with continuous variables). The corresponding field system was described by a Gaussian measure perturbed by a local interaction. At high temperatures the interaction was small and a suitable use of Brascamp–Lieb[5] inequalities allowed us to get a complete description of our system. The main point was that in this way we were also able to study the systems with (nonclassical) long-range interactions, i.e., these which are not absolutely summable. (In Section 2 of the present paper we give a generalization of the results of ref. 13 to systems with more complicated interactions).

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Let us note that the models of lattice spin systems with long-range interactions are considered to be important for understanding the physics of alloys of magnetic (e.g., Fe) with nonmagnetic (e.g., Au) metals. After studying the high-temperature disordered phase of interesting models, a challenging problem is to find a way to describe the low-temperature region. There is no rigorous result concerning the low-temperature behavior of systems with long-range interactions, aside from the mean-field Sherrington–Kirkpatrick models with random bond interactions[3] and with random sides.[14] In fact, even for systems with classical long-range interactions there is no systematic study of the low-temperature region except for ferromagnets (see, e.g., refs. 2, 9, 11, 15–17, and 19) and interactions with "small" long-range part.[22]

We mention also the results on the absence of phase transitions for random bond (classical) long-range interactions in refs. 4, 6–8, 10, and 21 and for classical long-range interactions in refs. 8, 12, 20, and 24. In the present paper we take a step in the direction of the low-temperature region for systems with long-range interactions. We argue that the field picture can be useful also for these pourposes.

First (in Section 3) we show that the spin picture and the field picture are equivalent in the sense that the sets of Gibbs measures of corresponding spin and field systems are isomorphic. Moreover, we prove that there is also a correspondence between order parameters, which allows us to give a meaning to the ferromagnetic and spin-glass phases in the field picture. To understand the low-temperature behavior of a lattice spin system, it is very important to know the structure of its ground states. Motivated by the results of Section 3, we propose to study this problem in the field picture. There we can deal with continuous variables, which makes the analysis possible. In this way we get some results concerning ground states in Section 4. In particular, we exhibit a class of systems possessing infinitely uncountably many ground states. One may expect that such systems can have an interesting behavior at low temperatures, including the spin-glass phase transition.

1. PRELIMINARIES

We consider a spin system on a lattice $\Gamma \equiv \mathbb{Z}^d$, $d \in \mathbb{N}$. Let $\mathscr{F}$ denote the family of finite sets in Γ. Let $\mathscr{F}_0 \equiv \{\Lambda_n \in \mathscr{F}\}_{n \in \mathbb{N}}$ be a countable base of $\mathscr{F}$, which means an increasing sequence of sets which is absorbing, i.e., for each $\Lambda \in \mathscr{F}$ there is $n_0 \in \mathbb{N}$ such that $\Lambda \subset \Lambda_{n_0}$. If not otherwise stated, $\mathscr{F}_0$ will be assumed to be a van Hove sequence. The number of elements in $\Lambda \in \mathscr{F}$ is denoted by $|\Lambda|$.

Let $(S, \mathscr{S})$ be a single spin space, consisting of the set $S = \{-1, 1\}$

and $\mathscr{S}$ the σ-algebra of subsets in S. A space of spin configurations s is defined to be $(\Omega, \Sigma) \equiv (S, \mathscr{S})^{\Gamma}$. A spin at site $i \in \Gamma$ is by definition a coordinate function

$$\Omega \ni s \mapsto s_i \in S \tag{1.1}$$

For $\Lambda \subset \Gamma$, let Σ_Λ be a σ-algebra generated by the functions $\{s_i : i \in \Lambda\}$. Let μ_0 be a free measure on (Ω, Σ) defined as a product of uniform probability measures on $(S; \mathscr{S})$. Similarly taking $O_0 \equiv \{0, 1\}$, we define a space $(O, \mathscr{O}) \equiv (O_0, \mathscr{O}_0)^{\Gamma}$. A coordinate function n_i, $i \in \Gamma$, on $(O, \mathscr{O})$ will be called an occupation number variable. Let $\mathscr{O}_\Lambda \subset \mathscr{O}$ with $\Lambda \subset \Gamma$ denote a σ-algebra generated by $\{n_i \colon i \in \Lambda\}$. We define the action of translation group on $(\Omega, \Sigma) \times (O, \mathscr{O})$ by setting

$$(T_k s)_i = s_{i-k}, \qquad (T_k n)_i := n_{i-k} \tag{1.2}$$

Using this, we can define a translation of functions and measures on $(\Omega, \Sigma) \times (O, \mathscr{O})$ in the usual way. Let E be a translation-invariant product probability measure on $(O, \mathscr{O})$. Let $\mathscr{M}$ denote the space of jointly measurable real functions on $(\Omega, \Sigma) \times (O, \mathscr{O})$. We wish to study the spin systems with interactions

$$\Phi \colon \quad \mathscr{F} \to \mathscr{M}$$

such that for any $X \in \mathscr{F}$, Φ_X is $\Sigma_X \times \mathscr{O}_X$ measurable (briefly, $\Phi_X \in \Sigma_X \times \mathscr{O}_X$) and

$$\Phi_X(n, \sigma) = n_X \Phi_X(\sigma) \tag{1.3}$$

where $n_X \equiv \prod_{i \in X} n_i$ and $\sigma_i \equiv n_i s_i$.

It is assumed that Φ can be represented in the form

$$\Phi = \theta + \phi \tag{1.4}$$

with θ a two-point interaction specified below and ϕ a classical Gibbsian interaction sattisfying

$$\|\phi\| \equiv \sup_{i \in \Gamma} \sum_{\substack{X \in \mathscr{F} \\ i \in X}} \sup_{\sigma} |\phi_X| < \infty \tag{1.5}$$

To define θ, we set $\theta_X \equiv 0$ for $|X| \neq 2$ and for $X = \{i, j\}$ we define

$$\theta_{i,j} := -\tfrac{1}{2} G_{i,j} \sigma_i \sigma_j \tag{1.6}$$

where

$$G_{ij} \equiv \frac{1}{(2\pi)^d} \int_{(-\pi, \pi)^d} d_d q \, e^{iq(i-j)} \, \hat{G}(q) \tag{1.7}$$

with $\hat{G}(q)$ a real, symmetric function satisfying for some $0<\varepsilon<\infty$

$$0<\varepsilon\leqslant\hat{G}(q)\leqslant\|\hat{G}\|_{\infty}<\infty \tag{1.8}$$

An interaction θ given by (1.6)–(1.8) can be a (nonclassical) long-range interaction in the sense that

$$\sum_{j\in\Gamma}|G_{ij}|=\infty \tag{1.9}$$

Nethertheless, as shown in refs. 13 and 23, the corresponding spin system behaves thermodynamically well. As an example of an interaction θ satisfying (1.9), we can take in one dimension

$$\hat{G}(q)\equiv\frac{\pi}{q_0}\chi(|q|<q_0) \tag{1.10}$$

for some $0<q_0<\pi$. Then

$$G_{ij}=\frac{\sin q_0\,|i-j|}{q_0\,|i-j|} \tag{1.11}$$

The other examples can be found in ref. 13 and together with some generalizations in ref. 23. For $\Lambda\in\mathscr{F}$ we define a Hamiltonian function by

$$H_\Lambda(\Phi)\equiv\sum_{X\subset\Lambda}\Phi_X \tag{1.12}$$

By our assumption (1.4) we can write it as follows:

$$H_\Lambda(\Phi)=H_\Lambda(\theta)+H_\Lambda(\phi) \tag{1.13}$$

2. SPIN AND FIELD PICTURES OF A LATTICE SYSTEM

We consider a measurable space $(\mathbb{R}^\Gamma,\mathscr{B})$, with $\mathscr{B}$ the Borel σ-algebra generated by product topology. Let μ_G be a Gaussian measure on $(\mathbb{R}^\Gamma,\mathscr{B})$ with mean zero and a covariance G given by (1.7)–(1.8).

Let φ_i, $i\in\Gamma$, denote the coordinate functions on $(\mathbb{R}^\Gamma,\mathscr{B})$. For $\beta\in\mathbb{R}^+$ and a Gibbsian interaction ϕ we define a finite-volume measure μ_Λ, $\Lambda\in\mathscr{F}$, on the space $(\mathbb{R}^\Gamma,\mathscr{B})\times(\Omega,\Sigma)$ by

$$\mu_\Lambda(F)\equiv\frac{\mu_G\otimes\mu_0(\{\exp(\beta^{1/2}\sum_{i\in\Lambda}\varphi_i\sigma_i)\exp[-\beta H_\Lambda(\phi(\sigma))]\}\,F(\varphi,\sigma))}{\mu_G\otimes\mu_0(\exp(\beta^{1/2}\sum_{i\in\Lambda}\varphi_i\sigma_i)\exp[-\beta H_\Lambda(\phi(\sigma))])} \tag{2.1}$$

By the definition we have

$$\mu_{\Lambda|\Sigma}(\cdot)=\mu_0(e^{-\beta H_\Lambda(\Phi)}\cdot)/\mu_0(e^{-\beta H_\Lambda(\Phi)}) \tag{2.2}$$

where on the rhs of (2.2) we recognize the usual finite-volume measure for a spin system with interaction Φ.

Let us introduce a function

$$U_\Lambda(\varphi):=\ln\left(\frac{\mu_0\exp[-\beta H_\Lambda(\phi)]\exp(\beta^{1/2}\sum_{i\in\Lambda}\varphi_i\sigma_i)}{\mu_0\exp[-\beta H_\Lambda(\phi)]}\right) \tag{2.3}$$

Then we have

$$\mu_{\Lambda|\mathscr{B}}(\cdot)=\mu_G(e^{U_\Lambda(\varphi)}\cdot)/\mu_G(e^{U_\Lambda(\varphi)}) \tag{2.4}$$

The important point is that $\mu_{\Lambda|\mathscr{B}}$ and $\mu_{\Lambda|\Sigma}$ determine each other. We have the following lemma essentially proven in ref. 13.

Lemma 2.1. Any expectation of polynomial functions in φ variables can be represented in terms of expectations of polynomial functions in σ variables and conversely. In particular, we have

$$\mu_\Lambda(\varphi_i)=\beta^{1/2}\sum_{j\in\Lambda}G_{ij}\mu_\Lambda\sigma_j \tag{2.5}$$

and

$$\mu_\Lambda\varphi_i\varphi_j=G_{ij}+\beta\sum_{k,k'\in\Lambda}G_{i,k}G_{j,k'}\mu_\Lambda\sigma_k\sigma_{k'} \tag{2.6}$$

To get a proof, one considers expectations of monomials in the field φ. Integrating by parts with the Gaussian measure μ_G, one gets a linear expression in terms of expectations of monomials in spin variables σ. Due to our assumption (1.8), this relation is invertible. It was observed in ref. 13 that, using the field picture, it is easier, and possible at all if one considers long-range θ interactions, to get a complete description of the corresponding spin system at high temperatures. Here we present a generalization of results of ref. 13. To formulate it, we shall introduce a finite-volume measure $\mu_\Lambda^{\tilde{\sigma}}$ with boundary conditions $\tilde{\sigma}$. We set

$$\mu_\Lambda^{\tilde{\sigma}}(\cdot)=\mu_\Lambda(e^{-\beta W_\Lambda(\tilde{\sigma},\phi)}\cdot)/\mu_\Lambda(e^{-\beta W_\Lambda(\tilde{\sigma},\phi)}) \tag{2.7}$$

with

$$W_\Lambda(\tilde{\sigma},\phi)\equiv-\sum_{i\in\Lambda}\sum_{j\in\Lambda^c}G_{ij}\tilde{\sigma}_j\sigma_i+\sum_{X\in\mathscr{F},X\cap\Lambda\neq\varnothing,X\cap\Lambda^c\neq\varnothing}\phi_X(\sigma_{X\cap\Lambda},\tilde{\sigma}_{X\cap\Lambda^c}) \tag{2.8}$$

whenever

$$\left| \sum_{j \in \Lambda^c} G_{ij} \tilde{\sigma}_j \right| < \infty \tag{2.9}$$

for all $\Lambda \in \mathscr{F}_0$ and $i \in \Lambda$. Let

$$U_\Lambda^{\tilde{\sigma}}(\varphi) := \lim_{\Lambda' \in \mathscr{F}_0} \ln \left[\delta_{\tilde{\sigma}} \frac{\mu_{0|\Sigma_\Lambda} \exp[-\beta H_{\Lambda'}(\phi)] \exp(\beta^{1/2} \sum_{j \in \Lambda} \varphi_i \sigma_j)}{\mu_{0|\Sigma_\Lambda} \exp[-\beta H_{\Lambda'}(\phi)]} \right] \tag{2.10}$$

with $\delta_{\tilde{\sigma}}$ a point measure concentrated on $\tilde{\sigma}$.

Theorem 2.2. For any interaction $\Phi \equiv \theta + \phi$, with ϕ a Gibbsian interaction satisfying (1.5) and θ given by (1.6)–(1.8), there is $0 < \beta_0 < \infty$ such that for any $0 < \beta < \beta_0$ the corresponding spin system is in the disordered phase. In particular, the Edwards–Anderson parameter

$$q_{EA} \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \sigma_i)^2 \tag{2.11}$$

is equal to zero and the system has a cluster property with the same decay as that of the interaction θ. Moreover, the limits of observables

$$\langle F \rangle \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} T_i F)^k \tag{2.12}$$

for $k \in \mathbb{N}$ and any local function F, exist and are independent of boundary condition $\tilde{\sigma}$.

REM: A similar result holds also for $O(N)$, $N \in \mathbb{N}$, models. Note that there is no loss of generality in assuming $0 < \varepsilon$ from (1.8) instead of $-\infty < \varepsilon$.

Proof. According to ref. 13, it is sufficient to pass to the field picture and then to show that there is

$$0 < \beta_0 < \|\hat{G}\|_\infty^{-1} \tag{2.13}$$

such that for all $0 < \beta < \beta_0$ the function

$$V_\Lambda(\varphi) \equiv -\left[\frac{\beta}{2} \sum_{i \in \Lambda} \varphi_i^2 - U_\Lambda^{\tilde{\sigma}}(\varphi) \right] \tag{2.14}$$

is concave for any $\Lambda \in \mathscr{F}$ and $\tilde{\sigma}$ satisfying (2.9). Using the definition (2.10) of $U_\Lambda^{\tilde{\sigma}}$, we get

$$\frac{\partial^2}{\partial_{\varphi_i} \partial_{\varphi_j}} V_\Lambda(\varphi) = -\beta[\delta_{ij} - \mu_{\Lambda, \beta(\phi + h(\varphi)\sigma)}^{\tilde{\sigma}}(\sigma_i, \sigma_j)] \tag{2.15}$$

where $\mu^{\tilde{\sigma}}_{\Lambda,\beta(\phi+h(\varphi)\sigma)}$ is a finite-volume spin measure corresponding formally to $\theta\equiv 0$ and Gibbsian interaction $\phi+h(\varphi)\sigma$ with

$$h(\varphi)\sigma=\{\beta^{-1/2}\varphi_i\sigma_i\} \tag{2.16}$$

Now by standard methods one can show that for sufficiently small β, the measure $\mu^{\tilde{\sigma}}_{\Lambda,\beta(\phi+h(\varphi)\sigma)}$ has a cluster property independent of volume $\Lambda\in\mathscr{F}$, "external magnetic field" $h(\varphi)$, and boundary conditions $\tilde{\sigma}$. Moreover, the quadratic form

$$\begin{aligned} A&\equiv\{A_{ij}\}\\ A_{ij}&\equiv\mu^{\tilde{\sigma}}_{\Lambda,\beta(\phi+h(\varphi)\sigma)}(\sigma_i,\sigma_j)\end{aligned} \tag{2.17}$$

satisfies

$$\|A\|\leqslant\sup_i\sum_j|\mu^{\tilde{\sigma}}_{\Lambda,\beta(\phi+h(\varphi)\sigma)}(\sigma_i,\sigma_j)|<1 \tag{2.18}$$

if $0<\beta<\beta_0$ for some sufficiently small β_0, $0<\beta_0<\infty$, independent of $\Lambda\in\mathscr{F}$, $\tilde{\sigma}$, and $h(\varphi)$. Therefore we have

$$\frac{\partial^2}{\partial_{\varphi_i}\partial_{\varphi_j}}V_\Lambda(\varphi)<0 \tag{2.19}$$

for all $\Lambda\in\mathscr{F}$ and $0<\beta<\beta_0$ with some $0<\beta_0<\infty$ sufficiently small. This allows as to apply the machinery of ref. 13 to get the statements of Proposition 2.2. ∎

Further, the field picture also can be used to consider low-temperature problems. For simplicity, from now now on we restrict ourselves to pure θ interactions plus eventual (random) external magnetic field. This case is sufficiently interesting and easier to study.

3. EQUIVALENCE OF SPIN AND FIELD PICTURES

Let θ be a two-point interaction defined in (1.6)–(1.8) and let $h\sigma\equiv\{h_in_is_i\}_{i\in\Gamma}$ be a one-point spin interaction. Let $\mu^{\tilde{\sigma}}_\Lambda$ denote the corresponding finite-volume measure (2.7) with external boundary condition given by configuration $\tilde{\sigma}$ [satisfying (2.9)]. For a field configuration $\tilde{\varphi}\in\mathbb{R}^\Gamma$ such that for any $\Lambda\in\mathscr{F}$

$$\left|\sum_{j\in\Lambda^c}G^{-1}_{ij}\tilde{\varphi}_j\right|<\infty \tag{3.1}$$

for all $i \in \Lambda$, we define a finite-volume measure $\pi_\Lambda^{\tilde{\varphi}}$ (with field external boundary conditions $\tilde{\varphi}$) by

$$\pi_\Lambda^{\tilde{\varphi}}(\cdot) \equiv \frac{1}{Z_\Lambda^{\tilde{\varphi}}} \int \prod_{i \in \Lambda} d\varphi_i \left[\exp\left(-\frac{1}{2} \sum_{i,j \in \Lambda} G_{ij}^{-1} \varphi_i \varphi_j - \sum_{\substack{j \in \Lambda^c \\ i \in \Lambda}} G_{ij}^{-1} \tilde{\varphi}_j \varphi_i \right) \times \exp(U_\Lambda) \cdot \right] \tag{3.2}$$

where $Z_\Lambda^{\tilde{\varphi}}$ is the normalization factor and

$$U_\Lambda(\varphi) \equiv \sum_{i \in \Lambda} \ln \frac{\mu_0(e^{-\beta h_i \sigma_i} e^{\beta^{1/2} \varphi_i \sigma_i})}{\mu_0(e^{-\beta h_i \sigma_i})} \tag{3.3}$$

One can see that for any $\Lambda' \in \mathscr{F}_0$ and $\Lambda \subset \Lambda'$ the kernel $\pi_\Lambda^{\tilde{\varphi}}$ agrees with the conditional expectation with respect to $\mathscr{B}_{\Lambda^c}$ and associated to the measure $\mu_{\Lambda'}$.

Now suppose we consider the measure $\mu_\Lambda^{\tilde{\sigma}}$. By the same arguments as used to prove Lemma 2.1, based on the integration by parts formula for Gaussian measure, one can see that the expectations $\mu_\Lambda^{\tilde{\sigma}}(W(\varphi))$ with local polynomials $W(\varphi)$ determine the expectations $\{\mu_\Lambda^{\tilde{\sigma}} \sigma_A\}_{A \in \mathscr{F}}$. This gives the following result.

Lemma 3.1. The infinite-volume field measure

$$\mu_{\mathscr{B}}^{\tilde{\sigma}} \equiv \lim_{\mathscr{F}_0} \mu_{\Lambda|\mathscr{B}}^{\tilde{\sigma}} \tag{3.4}$$

(if it exists) determines the spin measure

$$\mu_\Sigma^{\tilde{\sigma}} \equiv \lim_{\mathscr{F}_0} \mu_{\Lambda|\Sigma}^{\tilde{\sigma}} \tag{3.5}$$

Note that changing the integration variables

$$\varphi_i \mapsto \varphi_i' \equiv \varphi_i + \beta^{1/2} \sum_{j \in \Lambda^c} G_{ij} \tilde{\sigma}_j, \qquad i \in \Gamma \tag{3.6}$$

in expectations with the measure $\mu_\Lambda^{\tilde{\varphi}}$, we get for any local function $F \in \mathscr{B}_{\Lambda_0}$, $\Lambda_0 \in \mathscr{F}$, that

$$\mu_\Lambda^{\tilde{\sigma}} F(\varphi) = \lim_{\Lambda' \in \mathscr{F}_0} \frac{\mu_\Lambda[\exp(-\beta^{1/2} \sum_{j \in \Lambda^c \cap \Lambda'} \varphi_j \tilde{\sigma}_j)]\, F(\varphi - \beta^{1/2} \sum_{j \in \Lambda^c} G_{\cdot j} \tilde{\sigma}_j))}{\mu_\Lambda(\exp(-\beta^{1/2} \sum_{j \in \Lambda^c \cap \Lambda'} \varphi_j \tilde{\sigma}_j))} \tag{3.7}$$

This can be rewritten as follows:

$$\mu_\Lambda^{\tilde{\sigma}} F(\varphi) \equiv \rho_\Lambda^{\tilde{\sigma}} \left(F\left(\varphi - \beta^{1/2} \sum_{j \in \Lambda^c} G_{\cdot j} \tilde{\sigma}_j \right) \right) \tag{3.8}$$

with some probability measure $\rho_\Lambda^{\tilde{\sigma}}$.

Note that

$$\rho_\Lambda^{\tilde{\sigma}} F\left(\varphi - \beta^{1/2} \sum_{i \in \Lambda^c} G_{.j}\tilde{\sigma}_j\right) = \rho_\Lambda^{\tilde{\sigma}}\left(\pi_{\Lambda'}^{\varphi} F\left(\varphi - \beta^{1/2} \sum_{j \in \Lambda^c} G_{.j}\tilde{\sigma}_j\right)\right) \quad (3.9)$$

for any $\Lambda' \subset \Lambda$. Using this and the fact that locally

$$\lim_{\mathscr{F}_0} \sum_{j \in \Lambda^c} G_{ij}\tilde{\sigma}_j = 0 \quad (3.10)$$

formally, we get

$$\mu_{\mathscr{B}}^{\tilde{\sigma}} F(\varphi) \equiv \lim_{\mathscr{F}_0} \mu_\Lambda^{\tilde{\sigma}} F(\varphi) = \lim_{\mathscr{F}_0} \rho_\Lambda^{\tilde{\sigma}}\left(F\left(\varphi - \beta^{1/2} \sum_{j \in \Lambda^c} G_{.j}\tilde{\sigma}_j\right)\right)$$
$$= \lim_{\mathscr{F}_0} \rho_\Lambda^{\tilde{\sigma}}(F(\varphi)) \quad (3.11)$$

and

$$\lim_{\mathscr{F}_0} \rho_\Lambda^{\tilde{\sigma}}\left(F\left(\varphi - \beta^{1/2} \sum_{j \in \Lambda^c} G_{.j}\tilde{\sigma}_j\right)\right) = \lim_{\mathscr{F}_0} \rho_\Lambda^{\tilde{\sigma}}\left(\pi_{\Lambda'}^{\tilde{\varphi}}\left(F\left(\varphi - \beta^{1/2} \sum_{j \in \Lambda^c} G_{.j}\tilde{\sigma}_j\right)\right)\right)$$
$$= (\lim_{\mathscr{F}_0} \rho_\Lambda^{\tilde{\sigma}})(\pi_\Lambda^{\tilde{\varphi}}(F(\varphi))) \quad (3.12)$$

This implies that for any $\Lambda_0 \in \mathscr{F}$ the infinite-volume field measure $\mu_{\mathscr{B}}^{\tilde{\sigma}}$ satisfies

$$\mu_{\mathscr{B}}^{\tilde{\sigma}}(\pi_{\Lambda_0}^{\varphi}(F)) = \mu_{\mathscr{B}}^{\tilde{\sigma}}(F(\varphi)) \quad (3.13)$$

i.e., $\mu_{\mathscr{B}}^{\tilde{\sigma}}$ is a Gibbs measure for the family $\{\pi_\Lambda^{\varphi}\}_{\Lambda \in \mathscr{F}}$. This together with Lemma 3.1 give us the following result.

Theorem 3.2. Any infinite-volume spin measure $\mu_\Sigma^{\tilde{\sigma}}$ corresponding to the interaction $\{\theta + h\sigma\}$ is uniquely determined by the infinite-volume Gibbs measure $\mu_{\mathscr{B}}^{\tilde{\sigma}}$ of the corresponding field system.

In ref. 13 we saw that the use of the field picture proves to be a very effective tool to determine the properties of spin systems at high temperatures. (It turns out that it was easier to investigate the observable quantities

$$E\mu^{\tilde{\sigma}}(F) \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} T_i F)^k \quad (3.14)$$

or corresponding mean susceptibilities than the $\mu^{\tilde{\sigma}}$ themselves.)

Since Theorem 3.2 is independent of temperature, it may be useful for the study of the low-temperature region. Note that in the field picture we deal with continuous variables, which can make some renormalization group analysis possible. On the other hand, such an analysis in the case of spin variables is usually rather cumbersome. For the investigation of the low-temperature behavior of a lattice system it is helpful to have some information about its ground-state configurations minimizing the corresponding Hamiltonian function. It is very hard to get such information for a spin system with long-range interaction, as is usually the case when one deals with discrete variables. In the next section we show that such information is available for the field system. Therefore it may be better first to study the structure of the low-temperature field theory and then use Theorem 3.2 to get a description of the corresponding spin system.

To understand the physical content of this passage, we should give a meaning in the field picture phase transitions of interest to us, such as ferromagnetic and spin-glass phase transitions, which have a well-defined sense for spin systems. This is provided by the connection of the order parameters given below.

For $k = 1, 2$, let us define the following order parameters:

$$Q_\varphi^{(k)} \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \varphi_i)^k \tag{3.15}$$

and

$$Q_\sigma^{(k)} \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \sigma_i)^k \tag{3.16}$$

Theorem 3.3. The order parameters $Q_\varphi^{(k)}$ and $Q_\sigma^{(k)}$ are equivalent in the sense that

$$Q_\varphi^{(1)} = \beta^{1/2} \hat{G}(0) Q_\sigma^{(1)} \tag{3.17}$$

and

$$\beta \, \|\hat{G}^2\|^{-1} Q_\sigma^{(2)} \leqslant Q_\varphi^{(2)} \leqslant \beta \, \|\hat{G}^2\| \, Q_\sigma^{(2)} \tag{3.18}$$

Proof. Using the integration-by-parts formula for field variables, we get

$$\mu_\Lambda^{\tilde{\sigma}} \varphi_i = \beta^{1/2} \sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \tag{3.19}$$

Hence we have

$$\begin{aligned} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} \mu_\Lambda^{\tilde{\sigma}} \varphi_i &= \beta^{1/2} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} \sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \\ &= \beta^{1/2} \frac{1}{|\Lambda|} \sum_{i \in \Gamma} \sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \\ &\quad - \beta^{1/2} \frac{1}{|\Lambda|} \sum_{i \in \Lambda^c} \sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \end{aligned} \tag{3.20}$$

Since

$$\sum_{i \in \Gamma} G_{ij} = \hat{G}(0) \tag{3.21}$$

then

$$\lim_{\mathscr{F}_0} \beta^{1/2} \frac{1}{|\Lambda|} \sum_{i \in \Gamma} \sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j = \beta^{1/2} \hat{G}(0) Q_\sigma^{(1)} \tag{3.22}$$

On the other hand, we have formally

$$\lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda^c} \sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j = 0 \tag{3.23}$$

This together with (3.22) and (3.21) imply (3.17).

To show (3.18), we use (3.19) to get

$$\begin{aligned} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \varphi_i)^2 &= \beta \frac{1}{|\Lambda|} \sum_{i \in \Lambda} \left(\sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \right)^2 \\ &= \beta \frac{1}{|\Lambda|} \sum_{j, j' \in \Lambda} \left(\sum_{i \in \Lambda} G_{ij} G_{ij'} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \mu_\Lambda^{\tilde{\sigma}} \sigma_{j'} \right) \end{aligned} \tag{3.24}$$

Since

$$\sum_{i \in \Gamma} G_{ji} G_{ij'} = G_{jj'}^2 \tag{3.25}$$

then the rhs (3.24) can be written as follows:

$$\begin{aligned} \text{rhs}(3.24) = \beta \frac{1}{|\Lambda|} &\sum_{jj' \in \Lambda} G_{jj'}^2 \mu_\Lambda^{\tilde{\sigma}} \sigma_j \mu_\Lambda^{\tilde{\sigma}} \sigma_{j'} \\ &- \beta \frac{1}{|\Lambda|} \sum_{i \in \Lambda^c} \left(\sum_{j \in \Lambda} G_{ij} \mu_\Lambda^{\tilde{\sigma}} \sigma_j \right)^2 \end{aligned} \tag{3.26}$$

Hence

$$\frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \varphi_i)^2 \leqslant \beta \frac{1}{|\Lambda|} \sum_{j, j' \in \Lambda} G_{jj'}^2 \mu_\Lambda^{\tilde{\sigma}} \sigma_j \mu_\Lambda^{\tilde{\sigma}} \sigma_{j'} \tag{3.27}$$

Since from our assumption about $\hat{G}$ we have in the sense of quadratic forms

$$G_{ij}^2 \leqslant \|\hat{G}^2\|_\infty \delta_{ij} \tag{3.28}$$

then (3.27) implies that

$$\frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \varphi_i)^2 \leqslant \beta \, \|\hat{G}^2\|_\infty \frac{1}{|\Lambda|} \sum_{i \in \Lambda} (\mu_\Lambda^{\tilde{\sigma}} \sigma_i)^2 \tag{3.29}$$

Passing with Λ to Γ, we get the rhs inequality in (3.18). Inverting (3.19) and going by similar arguments as in (3.14)–(3.29), we get the rhs inequality of (3.18). This ends the proof of Theorem 3.3 ▌

4. ON THE STRUCTURE OF THE SET OF GROUND STATES FOR LATTICE SYSTEMS

In this section we study the structure of the set of ground states for the lattice systems considered in Section 3. Motivated by Theorem 3.2, we would like to do our analysis in the field picture, where we can deal with continuous variables. A field system of interest has the following formal action:

$$H(\varphi) \equiv \tfrac{1}{2} \sum_{i,j \in \Gamma} \varphi_i G_{ij}^{-1} \varphi_j - \sum_{i \in \Gamma} U_i(\varphi_i) \tag{4.1}$$

with

$$U_i(\varphi_i) \equiv \ln \left(\frac{\mu_0 e^{-\beta h_i \sigma_i} e^{\beta^{1/2} \varphi_i \sigma_i}}{\mu_0 e^{-\beta h_i \sigma_i}} \right)$$

$$\equiv \ln \operatorname{ch}[n_i(\beta^{1/2} \varphi_i - \beta h_i)] - \ln \operatorname{ch}(\beta h_i n_i) \tag{4.2}$$

The corresponding infinite-volume probability measure can be formally written as

$$\mu(F(\varphi)) \equiv \frac{1}{Z} \int \mathscr{D}\varphi \; e^{-H(\varphi)} F(\varphi) \tag{4.3}$$

The ground states of the field system are by definition equal to the global minima of the action H.

A necessary condition for φ to be a configuration minimalizing the action (4.1) reads

$$\frac{\partial}{\partial \varphi_i} H(\varphi) = \sum_{j \in \Gamma} G_{ij}^{-1} \varphi_j - U_i'(\varphi_i) = 0 \tag{4.4}$$

for all $i \in \Gamma$. Let us note that H is dependent on the reciprocal temperature β of the corresponding spin system. Therefore one may expect that also the set of ground states will depend on β. At high temperatures we have the following result.

Theorem 4.1. Suppose

$$\beta \, \|\hat{G}\| < 1 \tag{4.5}$$

Then for any configuration $\{n_i\}$ the action H has only one global minimum.

We remark that exactly under the condition (4.5) we have proven in ref. 13 that the corresponding lattice system is in the disordered phase.

Proof. It is sufficient to observe that under the condition (4.5), the Hessian form

$$\frac{\partial^2 H}{\partial\varphi_i \partial\varphi_j} \equiv G_{ij}^{-1} - \delta_{ij} \beta n_i \{\mathrm{ch}[n_i(-\beta h_i + \beta^{1/2}\varphi_i)]\}^{-2} \tag{4.6}$$

is strictly positive definite (no matter what the external magnetic field $\{h_i\}$ is). ∎

To analyze the low-temperature region, we use (4.2) and rewrite (4.4) in the more explicit form

$$(G^{-1}\varphi)_i = \beta^{1/2} n_i \,\mathrm{th}(-\beta h_i + \beta^{1/2}\varphi_i) \tag{4.7}$$

For simplicity we will take a translation-invariant external magnetic field $h_i = h$, $i \in \Gamma$.

Suppose that all $n_i \equiv 1$. Then a first easy solution of (4.7) (which exists for any G) is given by

$$\varphi_i = \xi_0, \qquad i \in \Gamma \tag{4.8}$$

where $\xi_0 \in \mathbb{R}$ satisfies

$$\left(\sum_{j \in \Gamma} G_{ij}^{-1} \right) \xi_0 = \beta^{1/2} \,\mathrm{th}(-\beta h + \beta^{1/2} \xi_0) \tag{4.9}$$

Using the fact that

$$\hat{G}_0^{-1} \equiv \sum_{j \in \Gamma} G_{ij}^{-1} > 0 \tag{4.10}$$

we introduce a new variable

$$\zeta_0 \equiv \beta^{-1/2} \hat{G}_0^{-1} \xi_0 \tag{4.11}$$

and write (4.9) as follows:

$$\zeta_0 = \text{th}[\beta\hat{G}_0(\zeta_0 - \hat{G}_0^{-1}h)] \tag{4.12}$$

Simple analysis of this equation gives the following result.

Theorem 4.2. If $h = 0$, then for $\beta\hat{G}_0 < 1$, Eq. (4.12) has only a zero solution (minimum of the action), whereas for $\beta\hat{G}_0 > 1$ there are two nontrivial solutions $\pm|\zeta_0|$ corresponding to minima of the action and the zero solution corresponding to a local maximum. Suppose $h \neq 0$. If $|\hat{G}_0^{-1}h| < 1$, then there is $\beta_c \equiv \beta_c(h)$ satisfying $\beta_c\hat{G}_0 > 1$ such that for any $\beta > \beta_c$, Eq. (4.12) has three solutions $\zeta_1 < \zeta_2 < \zeta_3$. Two of them, ζ_2 and ζ_3, correspond to minima of the action and ζ_2 to a local maximum. There is only one global minimum of the action.

For $|\hat{G}_0^{-1}h| > 1$, Eq. (4.12) has only one solution, no matter how small the temperature.

It is useful to introduce the energy density $e(\varphi)$ of a ground state φ by

$$e(\varphi) \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} H(\varphi_{|\Lambda}) \tag{4.13}$$

with $\varphi_{|\Lambda}$ coinciding with φ inside Λ and identically equal to zero outside Λ. In the translation-invariant case for φ given by (4.8)–(4.12) we get

$$e(\varphi) = \tfrac{1}{2}\hat{G}_0^{-1}\xi_0^2 - \ln \text{ch}(-\beta h + \beta^{1/2}\xi_0) \tag{4.14}$$

Now let us consider a non-translation-invariant case. Assuming that $h_i \equiv 0$, $i \in \Gamma$, we would like to look for ground states of the form

$$\varphi_i = \xi\tilde{\sigma}_i \tag{4.15}$$

where $|\xi| \geqslant 0$ and $\tilde{\sigma}_i \equiv n_i s_i$ satisfies

$$\sum_{j\in\Gamma} G_{ij}^{-1}\tilde{\sigma}_j = \lambda^{-1}\tilde{\sigma}_i \tag{4.16}$$

with some $\lambda \in \mathbb{R}^+$, $0 \leqslant \lambda^{-1} \leqslant \|\hat{G}\|^{-1}$.

Using (4.7), (4.15), and (4.16) we get the following equation for ξ:

$$\lambda^{-1}\xi = \beta^{1/2}\,\text{th}(\beta^{1/2}\xi) \tag{4.17}$$

Introducing

$$\zeta \equiv \beta^{-1/2}\lambda^{-1}\xi \tag{4.18}$$

we can rewrite (4.17) in the form

$$\zeta = \text{th}(\beta\lambda\zeta) \tag{4.19}$$

It is known that (4.19) has nonzero solution iff

$$\beta\lambda > 1 \tag{4.20}$$

For such β we have two nonzero solutions differing only in sign.

Let us now come back to condition (4.16), which is the main point of our analysis. We would like to exhibit a class of G for which (4.16) is satisfied for infinitely uncountably many configurations $\{\tilde{\sigma}_i \equiv n_i s_i\}$.

Definition 4.3. We say that a covariance G satisfies a condition (C) iff there is an open symmetric set $A_0 \subset \operatorname{supp} G$ such that for any $q \in A_0$

$$\hat{G}(q) = \lambda = \text{const} \tag{4.21}$$

Note that condition (C) defines a class containing interesting examples of long-range interactions, e.g., given by (1.10).

Let $\{\mu\}$ now be a family of probability measures on (Ω, Σ) dependent on $n \in O$ satisfying for

$$C_{ij} := E\mu(\tilde{\sigma}_i \tilde{\sigma}_j) \tag{4.22a}$$

$$C_{ij} \equiv \frac{1}{(2\pi)^d} \int_{(-\pi,\pi)^d} d_d q\, e^{iq(i-j)}\, \hat{C}(q) \tag{4.22b}$$

the requirements

$$\operatorname{supp} \hat{C}(q) \equiv A \subseteq A_0 \tag{4.23a}$$

and

$$\hat{C}(q) = (2\pi)^d c \chi(q \in A) \tag{4.23b}$$

with some constant $0 < c < \infty$.

Note that

$$E\mu(\tilde{\sigma}_i \tilde{\sigma}_i) = C_{ii} = c\,|A| \tag{4.24}$$

and since $\tilde{\sigma}_i \tilde{\sigma}_i = n_i$, so c satisfies the normalization condition

$$c = \frac{E(n_0)}{|A|} \tag{4.25}$$

Using the conditions (4.21)–(4.23), we see that

$$E\mu\left(\sum_{j \in \Gamma} G_{ij}^{-1} \tilde{\sigma}_j - \lambda^{-1} \tilde{\sigma}_i\right)^2 = 0 \tag{4.26}$$

This is because the lhs of (4.26) equals

$$\sum_{j,j'} G_{ij}^{-1} C_{jj'} G_{j'i}^{-1} - 2\lambda^{-1} \sum_{j} G_{ij}^{-1} C_{ji} + \lambda^{-2} C_{ii}$$

$$= \frac{1}{(2\pi)^d} \int_{(-\pi,\pi)^d} d_d q \, [\hat{G}(q)^{-2} \, \hat{C}(q) - 2\lambda^{-1} \hat{G}(q)^{-1} \hat{C}(q) + \lambda^{-2} \hat{C}(q)]$$

$$= \frac{1}{(2\pi)^d} \int_{(-\pi,\pi)^d} d_d q \, [\hat{G}(q)^{-1} - \lambda^{-1}]^2 \, \hat{C}(q) \tag{4.27}$$

and by our assumptions

$$\hat{G}(q)^{-1} - \lambda^{-1} = 0 \tag{4.28}$$

on the support of $\hat{C}(q)$.

The equality (4.26) means that there is a set $\tilde{O}$ of E of measure one and a set of measures $\{\mu_n : n \in \tilde{O}\}$ such that (4.16) is satisfied μ_n-a.e., $n \in \tilde{O}$.

To finish our considerations, we only mention that using the methods of ref. 13 one can really construct a family of continuous measures $\{\mu_n\}$, $n \in O$, satisfying (4.22) and (4.23).

Suppose we have $\tilde{\sigma}$ and $\xi \neq 0$ satisfying (4.16) and (4.17). We want to show that the corresponding φ defined by (4.15) is a minimum of our action if β is sufficiently big. To see that, we consider the Hessian

$$\frac{\partial^2 H}{\partial \varphi_i \, \partial \varphi_j} (\varphi) = G_{ij}^{-1} - \delta_{ij} \, \beta n_i (\text{ch} \, \beta^{1/2} \xi \tilde{\sigma}_j)^{-2}$$

$$= G_{ij}^{-1} - \delta_{ij} n_i \beta (\text{ch} \, \beta \lambda \zeta n_i)^{-2} \tag{4.29}$$

where $\zeta \equiv \zeta(\beta)$ satisfies (4.19). If $\beta \to \infty$, then $\zeta(\beta) \to 1$. Therefore, there is $0 < \beta_c < \infty$ such that

$$\beta (\text{ch} \, \beta \lambda \zeta)^{-2} < \|\hat{G}\|^{-1} \tag{4.30}$$

for all $\beta > \beta_c$. This, however, implies

$$\frac{\partial^2 H}{\partial \varphi_i \, \partial \varphi_j} (\varphi) > 0 \tag{4.31}$$

for all $\beta > \beta_c$, i.e., corresponding configurations are the ground states of our system. Note that, as follows from (4.29), the dilution improves the properties of the system, in the sense that its action becomes more and more convex.

Let us compute formally the corresponding energy density, defined by

$$e(\varphi) \equiv \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} H(\varphi_{|\Lambda}) \tag{4.32}$$

with $\varphi_{|\Lambda}$ being equal to φ in Λ and zero outside. We have

$$\frac{1}{|\Lambda|} H(\varphi_{|\Lambda}) = \frac{1}{|\Lambda|} \left(\frac{1}{2} \xi^2 \sum_{i,j \in \Lambda} G_{ij}^{-1} \tilde{\sigma}_i \tilde{\sigma}_j - \sum_{i \in \Lambda} \ln \operatorname{ch} \beta^{1/2} \xi \tilde{\sigma}_i \right)$$
$$= \frac{1}{|\Lambda|} \left(\frac{1}{2} \xi^2 \lambda^{-1} \sum_{i \in \Lambda} \tilde{\sigma}_i^2 - \sum_{i \in \Lambda} \ln \operatorname{ch} \beta^{1/2} \xi \tilde{\sigma}_i \right)$$
$$- \frac{1}{|\Lambda|} \frac{1}{2} \xi^2 \sum_{\substack{i \in \Lambda \\ j \in \Lambda^c}} G_{ij}^{-1} \tilde{\sigma}_i \tilde{\sigma}_j \tag{4.33}$$

The second term on the rhs of (4.33) converges formally to zero as $\Lambda \uparrow \Gamma$. The first term on the rhs of (4.33) in the limit gives

$$\lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} \left(\frac{1}{2} \xi^2 \lambda^{-1} \tilde{\sigma}_i^2 - \ln \operatorname{ch} \beta^{1/2} \xi \tilde{\sigma}_i \right)$$
$$= \left(\frac{1}{2} \xi^2 \lambda^{-1} - \ln \operatorname{ch} \beta^{1/2} \xi \right) \lim_{\mathscr{F}_0} \frac{1}{|\Lambda|} \sum_{i \in \Lambda} n_i$$
$$= \left(\frac{1}{2} \xi^2 \lambda^{-1} - \ln \operatorname{ch} \beta^{1/2} \xi \right) E n_0 \tag{4.34}$$

where we used the fact that E is a translation-invariant ergodic measure. Let us remark that

$$(\tfrac{1}{2} \xi^2 \lambda^{-1} - \ln \operatorname{ch} \beta^{1/2} \xi)\, E n_0 = (\tfrac{1}{2} \beta \lambda \zeta^2 - \ln \operatorname{ch} \beta \lambda \zeta)\, E n_0 < 0 \tag{4.35}$$

for large β. For given $\hat{G}$ satisfying (C) any family $\{\mu_n\}$ for which (4.22) and (4.23) holds gives the same result.

Summarizing, we have the following result.

Theorem 4.4. Suppose $h = 0$ and $\hat{G}$ has a flat piece, i.e., there is an open set $A_0 \subset\subset (-\pi, \pi)^d$ such that

$$\hat{G}(q) = \lambda \equiv \max \hat{G}(q)$$

for all $q \in A_0$. Then the corresponding spin system possesses infinitely uncountably many ground states given by (4.15)–(4.17), with the same energy density

$$e(\varphi) = (\tfrac{1}{2} \lambda^{-1} \xi^2 - \ln \operatorname{ch} \beta^{1/2} \xi)\, E n_0 \tag{4.36}$$

Moreover, each ground state φ is spin-flip degenerate; i.e., $(-\varphi)$ is also a ground state.

6. M. Campanino, E. Oliveri, and A. C. D. van Enter, One dimensional spin glasses with potential decay $r^{-(1+\varepsilon)}$. Absence of phase transitions and cluster properties, *Commun. Math. Phys.* **108**:241–255 (1987).
7. M. Campanino and E. Oliveri, One-dimensional random Ising systems with interaction decay $r^{-(1+\varepsilon)}$. A convergent cluster expansion, *Commun. Math. Phys.* **111**:555–577 (1987).
8. M. Cassandro, E. Oliveri, and B. Tirozzi, Infinite differentiability for one-dimensional spin systems with long range interaction, *Commun. Math. Phys.* **87**:229–252 (1982).
9. F. J. Dyson, Existence of a phase transition in a one-dimensional Ising ferromagnet, *Commun. Math. Phys.* **12**:91–107 (1969).
10. A. C. D. van Enter, and J. Fröhlich, Absence of symmetry breaking for N-vector spin glass models in two dimensions, *Commun. Math. Phys.* **98**:425–432 (1985).
11. J. Fröhlich and T. Spencer, The phase transition in one-dimensional Ising model with $1/r^2$ interaction energy, *Commun. Math. Phys.* **84**:87–101 (1982).
12. J. Fröhlich and C.-E. Pfister, On the absence of spontanous symmetry breaking and of crystalline ordering in two-dimensional systems, *Commun. Math. Phys.* **81**:277–298 (1981).
13. J. Fröhlich and B. Zegarlinski, Spin glasses and other lattice systems with long range interactions, *Commun. Math. Phys.* **120**:665–688 (1989).
14. J. L. van Hemmen, Equilibrium theory of spin glasses: Mean field theory and beyound, in *Heidelberg Colloquium on Spin Glasses*, J. L. van Hemmen and I. Morgenstern, eds. (Lecture Notes in Physics, 192, Springer-Verlag, 1983), pp. 203–233.
15. J. Z. Imbrie, Decay of correlation in the one-dimensional Ising model with $J_{ij} = |i-j|^{-2}$, *Commun. Math. Phys.* **85**:491–516 (1982).
16. J. Z. Imbrie and C. M. Newman, An intermediate phase with slow decay of correlations in one-dimensional $1/|x-y|^2$ percolation, Ising and Potts models, *Commun. Math. Phys.* **118**:303 (1969).
17. R. B. Israel, *Convexity in the Theory of Lattice Gases* (Princeton University Press, 1979).
18. K. M. Khanin, Absence of phase transitions in one-dimensional long-range spin systems with random Hamiltonian, *Theor. Math. Phys.* **43**:445 –449 (1980).
19. H. Kunz and C.-E. Pfister, First order phase transition in the plane rotator feromagnetic model in two dimensions, *Commun. Math. Phys.* **46**:245–251 (1976).
20. C.-E. Pfister, On the symmetry of the Gibbs states in two dimensional lattice systems, *Commun. Math. Phys.* **79**:181–188 (1981).
21. P. Picco, On the absence of breakdown of symmetry for the plane rotator model with long-range interaction, *J. Stat. Phys.* **32**:627–648 (1983).
22. Y. M. Park, Extension of Pirogov–Sinai theory of phase transitions to infinite range interactions. I. Cluster expansion, *Commun. Math. Phys.* **114**:197–218 (1988); II. Phase diagram, *Commun. Math. Phys.* **114**:219–241 (1988).
23. B. Zegarlinski, Interactions and pressure functional for disordered lattice systems, Preprint, Bochum (1989).
24. R. L. Dobrushin and S. B. Shlosman, Absence of breakdown of continuous symmetry in two-dimensional models of statistical physics, *Commun. Math. Phys.* **42**:31–40 (1975).